- **I can complete coordinate proofs.**

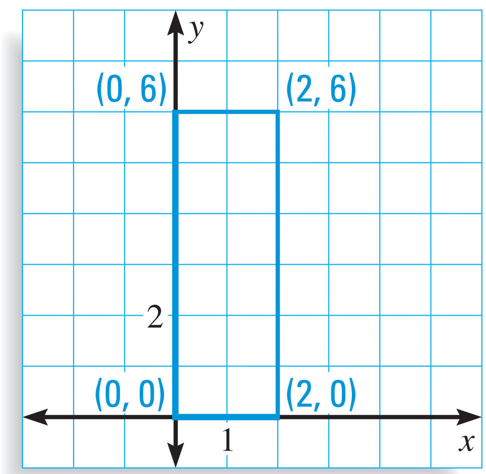
So far, you have studied two-column proofs, paragraph proofs, and flow proofs. A **coordinate proof** involves placing geometric figures in a coordinate plane. Then you can use the Distance Formula and the Midpoint Formula, as well as postulates and theorems, to prove statements about the figures.

EXAMPLE 1 *Placing a Rectangle in a Coordinate Plane*

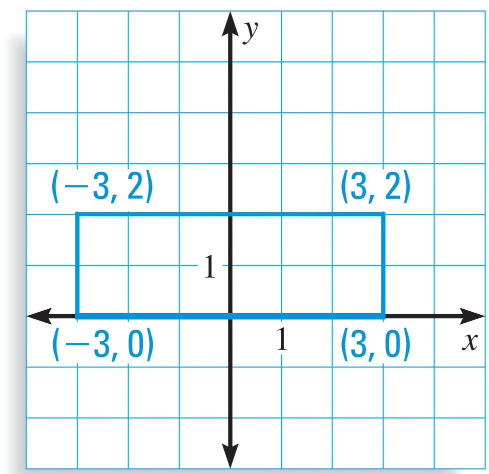
Place a 2-unit by 6-unit rectangle in a coordinate plane.

SOLUTION

Choose a placement that makes finding distances easy. Here are two possible placements.



One vertex is at the origin, and three of the vertices have at least one coordinate that is 0.

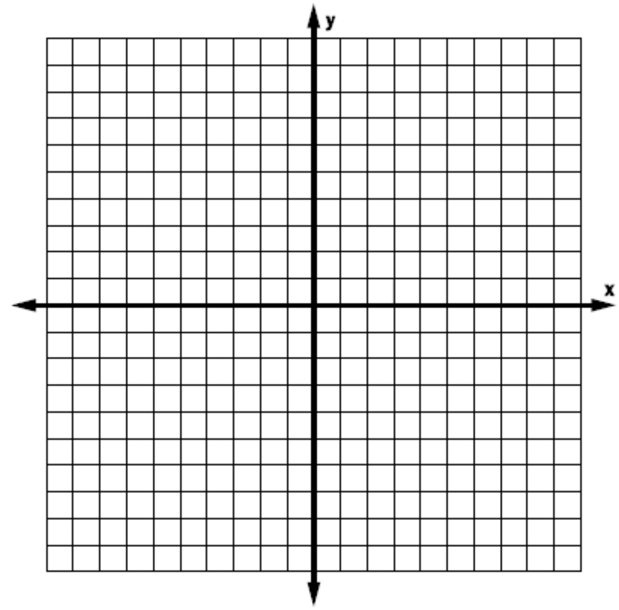


One side is centered at the origin, and the *x*-coordinates are opposites.

Pre-AP Geometry Date_____ 4.7 Notes

Triangles and Coordinate Proof (pp 243-246)

1. Place a 3-unit by 5-unit rectangle in a coordinate plane.



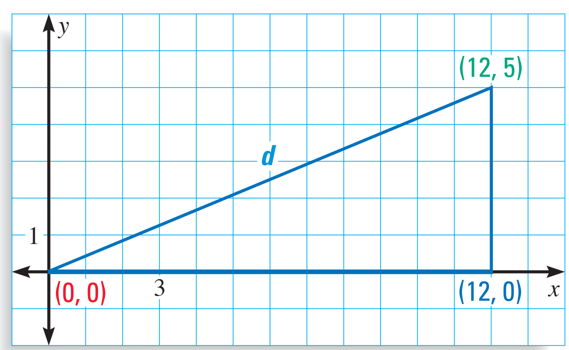
EXAMPLE 2 *Using the Distance Formula*

Using Algebra

A right triangle has legs of 5 units and 12 units. Place the triangle in a coordinate plane. Label the coordinates of the vertices and find the length of the hypotenuse.

Solution

One possible placement is shown. Notice that one leg is vertical and the other leg is horizontal, which assures that the legs meet at right angles. Points on the same vertical segment have the same x-coordinate, and points on the same horizontal segment have the same y-coordinate.



You can use the Distance Formula to find the length of the hypotenuse.

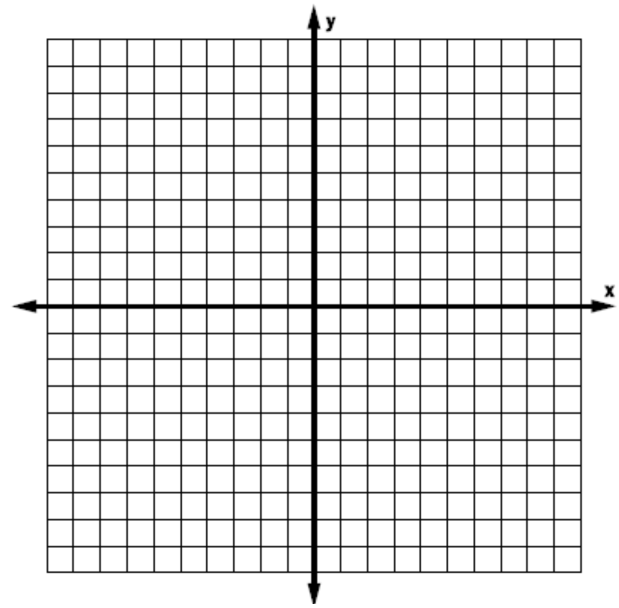
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ **Distance Formula**

$= \sqrt{(12 - 0)^2 + (5 - 0)^2}$ **Substitute.**

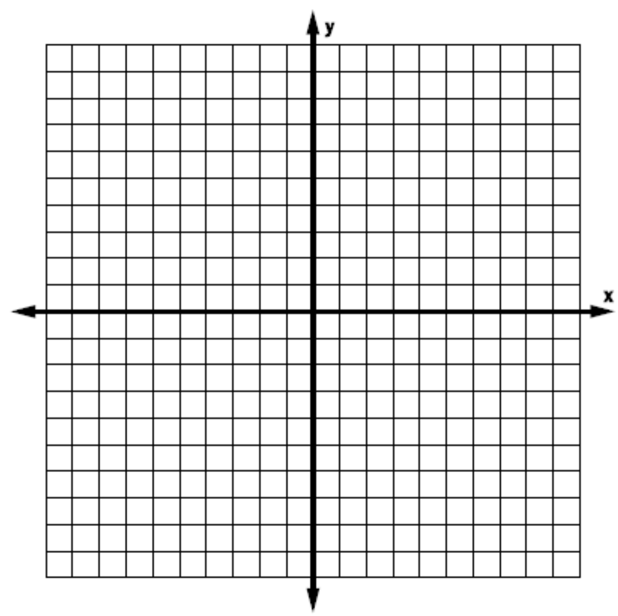
$= \sqrt{169}$ **Simplify.**

$= 13$ **Evaluate square root.**

2. A right triangle has legs of 6 and 8. Place the triangle in a coordinate plane. Label the vertices and find the length of the hypotenuse.



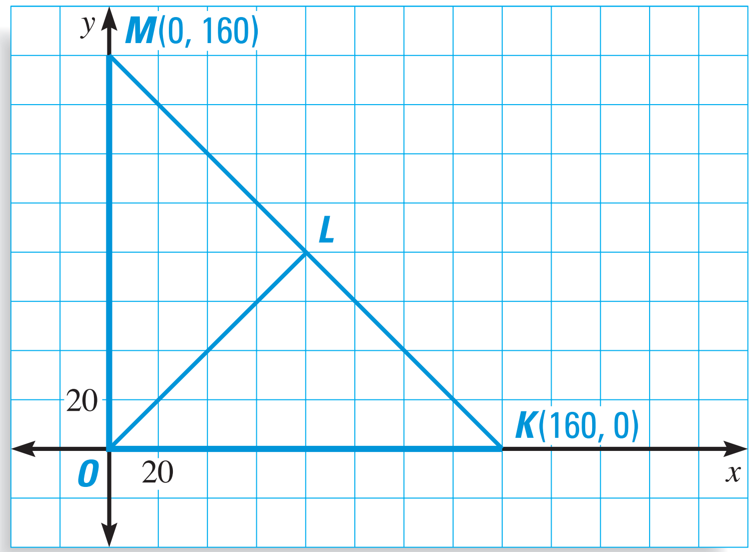
3. A right triangle has legs of 2.5 and 6. Place the triangle in a coordinate plane. Label the vertices and find the length of the hypotenuse.



EXAMPLE 3 *Using the Midpoint Formula*

In the diagram, $\triangle MLO \cong \triangle KLO$.

Find the coordinates of point L.



SOLUTION

Because the triangles are congruent, it follows that $\overline{ML} \cong \overline{KL}$. So, point L must be the midpoint of $\overline{MK}$. This means you can use the Midpoint Formula to find the coordinates of point L.

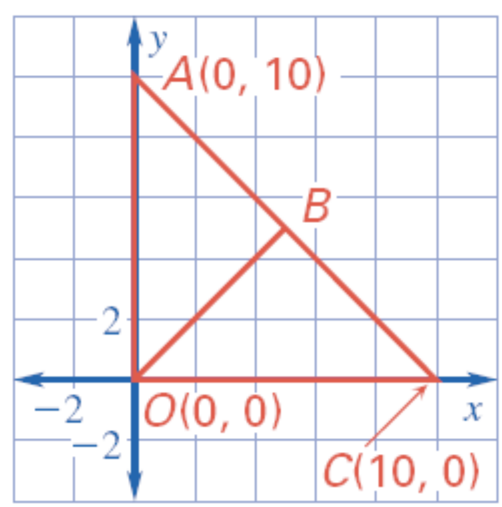
$$L(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$ **Midpoint Formula**

$$= \left(\frac{160 + 0}{2}, \frac{0 + 160}{2}\right)$$ **Substitute.**

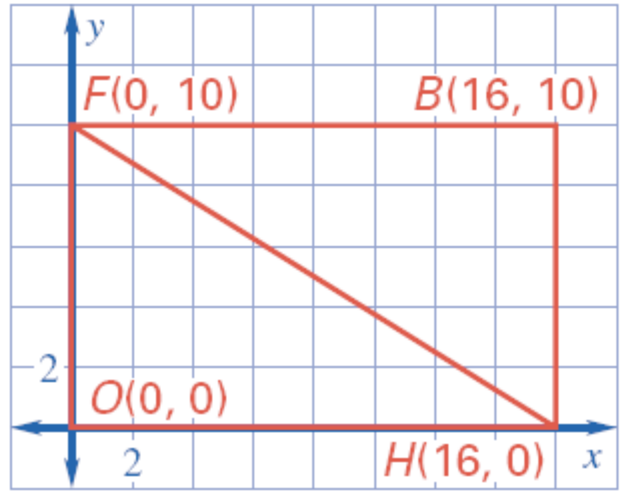
$$= (80, 80)$$ **Simplify.**

▶ The coordinates of L are (80, 80).

4. In the diagram, $\triangle ABO \cong \triangle CBO$. Find the coordinates of point B.



5. Guided Practice: $\triangle FHB \cong \triangle HFO$. Find the coordinates of R if R is the midpoint of $\overline{FH}$.



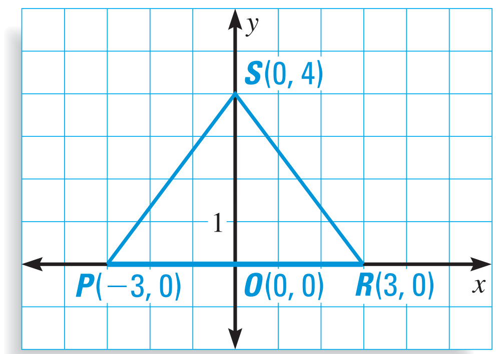
EXAMPLE 4 ***Writing a Plan for a Coordinate Proof***

Proof

Write a plan to prove that $\overrightarrow{SO}$ bisects $\angle PSR$.

GIVEN ▶ Coordinates of vertices of $\triangle POS$ and $\triangle ROS$

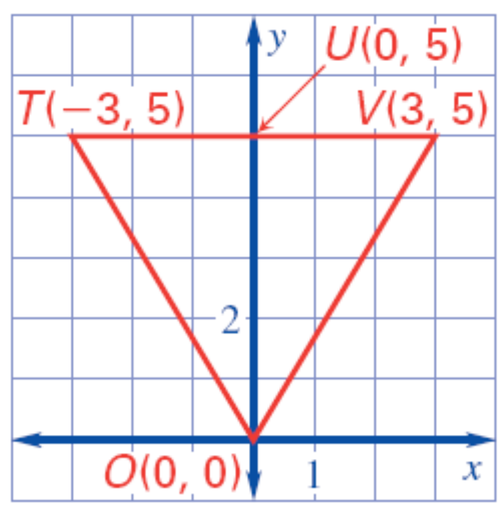
PROVE ▶ $\overrightarrow{SO}$ bisects $\angle PSR$



SOLUTION

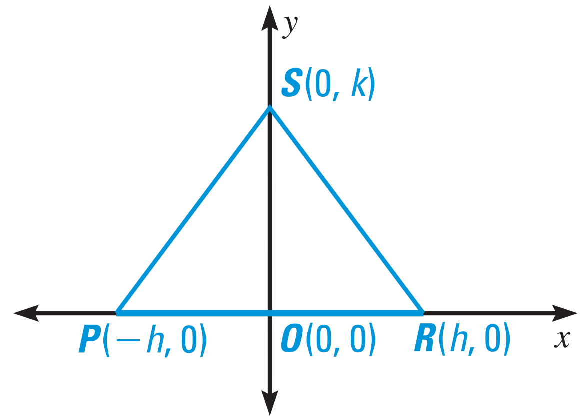
Plan for Proof Use the Distance Formula to find the side lengths of $\triangle POS$ and $\triangle ROS$. Then use the SSS Congruence Postulate to show that $\triangle POS \cong \triangle ROS$. Finally, use the fact that corresponding parts of congruent triangles are congruent to conclude that $\angle PSO \cong \angle RSO$, which implies that $\overrightarrow{SO}$ bisects $\angle PSR$.

6. Write a plan to prove that $\overline{OU}$ bisects $\measuredangle TOV$.



The coordinate proof in Example 4 applies to a specific triangle. When you want to prove a statement about a more general set of figures, it is helpful to use variables as coordinates.

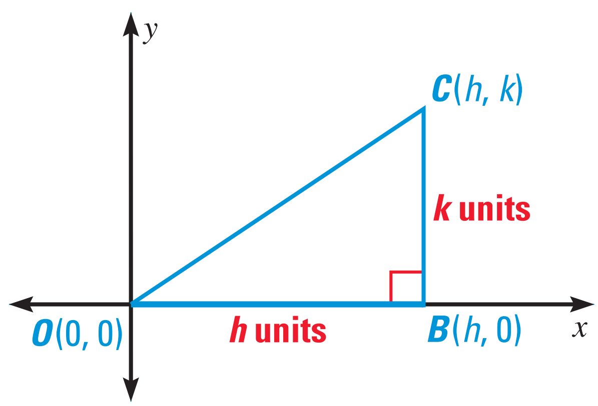
For instance, you can use variable coordinates to duplicate the proof in Example 4. Once this is done, you can conclude that $\overrightarrow{SO}$ bisects $\angle PSR$ for *any* triangle whose coordinates fit the given pattern.



EXAMPLE 5 *Using Variables as Coordinates*

Right $\triangle OBC$ has leg lengths of h units and k units. You can find the coordinates of points B and C by considering how the triangle is placed in the coordinate plane.

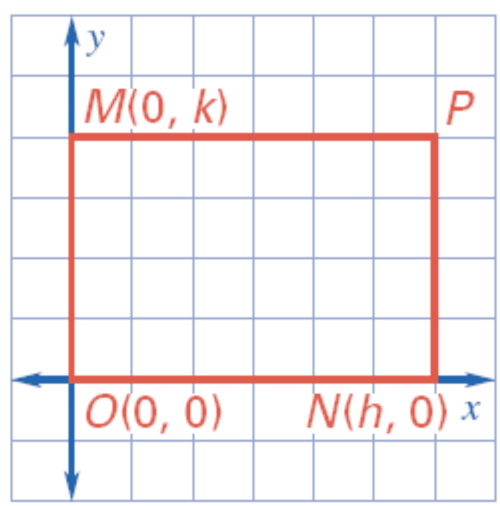
Point B is h units horizontally from the origin, so its coordinates are $(h, 0)$. Point C is h units horizontally from the origin and k units vertically from the origin, so its coordinates are (h, k).



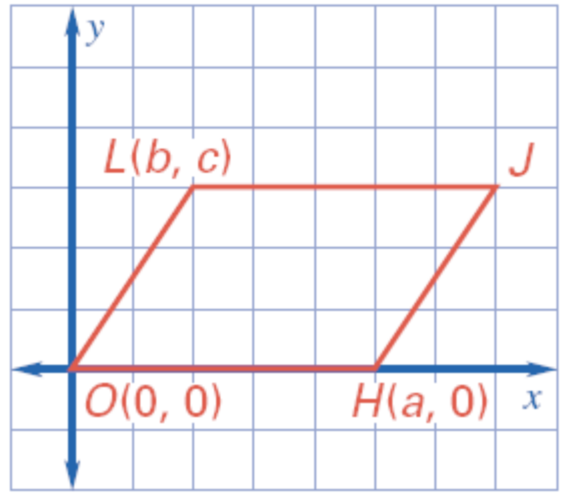
You can use the Distance Formula to find the length of the hypotenuse $\overline{OC}$.

$$OC = \sqrt{(h-0)^2 + (k-0)^2} = \sqrt{h^2 + k^2}$$

7. Find the coordinates of P.



8. Find the coordinates of *J*.

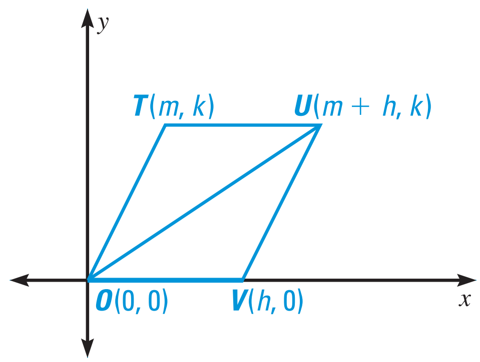


EXAMPLE 6 *Writing a Coordinate Proof*

Proof

GIVEN ▶ Coordinates of figure $OTUV$

PROVE ▶ $\triangle OTU \cong \triangle UVO$



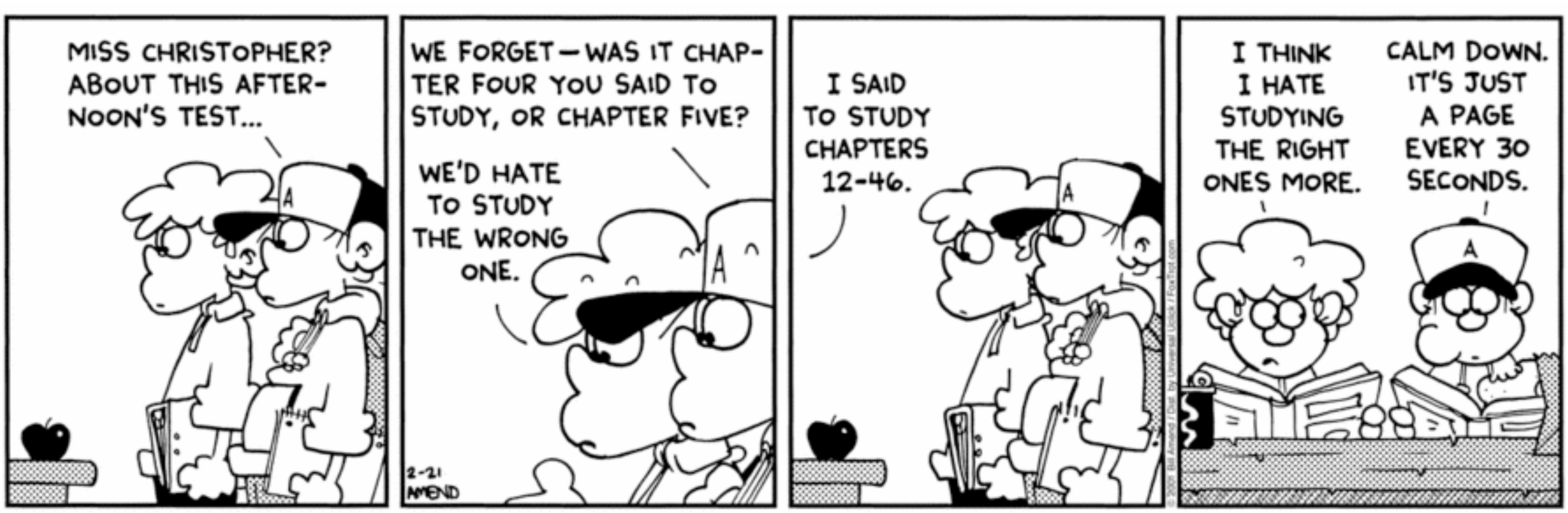
SOLUTION

COORDINATE PROOF Segments $\overline{OV}$ and $\overline{UT}$ have the same length.

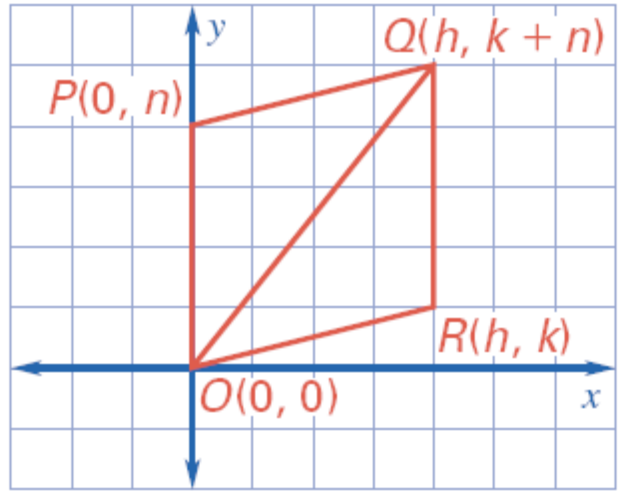
$$OV = \sqrt{(h-0)^2 + (0-0)^2} = h$$

$$UT = \sqrt{(m+h-m)^2 + (k-k)^2} = h$$

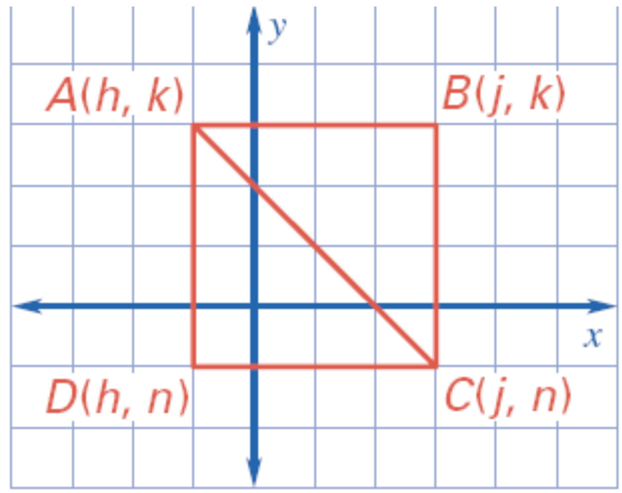
Horizontal segments $\overline{UT}$ and $\overline{OV}$ each have a slope of 0, which implies that they are parallel. Segment $\overline{OU}$ intersects $\overline{UT}$ and $\overline{OV}$ to form congruent alternate interior angles $\angle TUO$ and $\angle VOU$. Because $\overline{OU} \cong \overline{OU}$, you can apply the SAS Congruence Postulate to conclude that $\triangle OTU \cong \triangle UVO$.



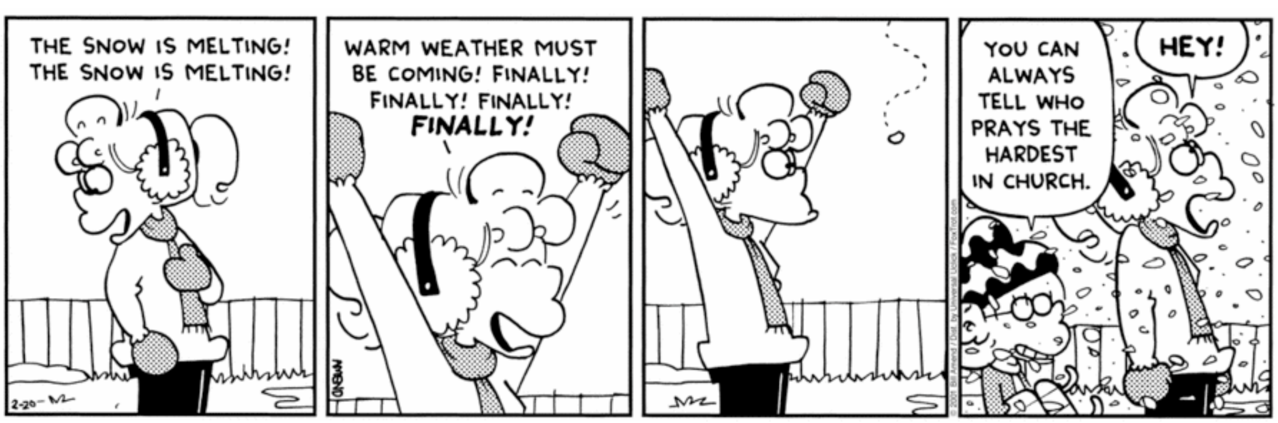
9. Prove: $\triangle OPQ \cong \triangle QRO$



10. Prove: $\triangle ABC \cong \triangle CDA$



11. State the similairities and differences between a coordinate proof and other types of proofs.



Pre-AP Geometry Date_____ 4.7 Notes

Triangles and Coordinate Proof (pp 243-246)

12. Two different ways to place the same right triangle in a coordinate plane are shown. Which placement is more convenient for finding the side lengths? Explain your thinking. Then sketch a third placement that also makes it convenient to find the side lengths.